Application of the Bubble-Check algorithm to Non-Binary LLR computation in QAM coded schemes

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This Letter considers the generation of intrinsic LLR messages in non-binary coded schemes associated to QAM modulation. We show that intrinsic LLR message generation corresponds to the same kind of computation than the one performed at the elementary check nodes in Extended Min-Sum non-binary LDPC decoders, i.e. finding a given number of minimum values in a structured set. We propose to use the Bubble-Check algorithm for the LLR calculation to benefit from two advantages: low-complexity hardware architecture and sharing the same hardware at the demapping and the decoding step.

Introduction: Non-binary (NB) coded schemes can be naturally associated to high-order modulation for high data rate, leading to low-error high-spectral-efficiency communication systems. Compared to binary coded schemes, the use of NB codes improves the performance of the decoding algorithms [1] [2] as the intrinsic likelihoods of the received symbols (which are the inputs to the decoder) are uncorrelated from one symbol to another. Recent work on QAM binary soft demapping include [3].

Figure 1 presents the schematics of the considered digital communication chain. Information data are encoded by a NB encoder and then mapped to a symbol in the QAM constellation. Note that the order of the NB code, q, corresponds to the number of signals in the modulation, M (i.e. $M = q$). After the channel, the modulated noisy symbols are demapped to generate the intrinsic message. Note that this Letter focusses on the demapper block and considers the generation of the intrinsic likelihood messages. Even if our approach may be considered for any NB coded modulation scheme that needs the sorting of elements in the likelihood vector, we essentially focus on NB-LDPC and Extended Min-Sum (EMS) [4] decoding algorithms [1].

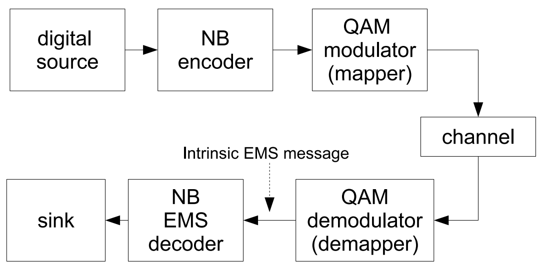

Fig. 1. Digital communication chain

NB coded modulation: Let $\mathbf{X} = (x_1, x_2, \ldots, x_N)$ be the codeword generated by the NB encoder, where x_k is an element of GF(q), i.e. $x_k = a_p, p = 0, \ldots, q-1$. Let C be the mapping of the set GF(q) to the set of points of the constellation (each point represents a modulation signal):
$C : a_p \in \text{GF}(q) \to (\pi_I(p), \pi_Q(p))_{p=0,1,2,\ldots,q-1} \in M\text{-QAM}$.
In other words, the 2^m-ary QAM (where m is even) is decomposed into two independent $2^{m/2}$-ary Pulse Amplitude Modulations (PAMs). For each a_p, the I coordinate corresponds to the in-phase axis (*idem* Q coordinate, in-quadrature axis).

The received noisy codeword $\mathbf{Y}$ consists of N NB symbols independently affected by noise. Each symbol is represented by $y_k = C(x_k) + w_k, k \in \{1, 2, ..., N\}$, w_k is the realization of a complex Additive White Gaussian Noise (AWGN) of variance σ^2.

The EMS decoding algorithm: The EMS algorithm was proposed for NB-LDPC low-complexity decoding in [4] [6] as a generalization of the Min-Sum algorithm used for binary LDPC codes ([7], [8] and [9]). Its principle is the truncation of the vector messages from q to n_m values ($n_m << q$). The complexity/performance trade-off can be adjusted with the value of the n_m parameter. This characteristic makes the EMS decoder architecture easily adaptable to both implementation and performance constraints.

[1] the Min-Max [5] decoding algorithm would also fit in our study

The EMS intrinsic message: For each received symbol y_k the intrisic message is composed of n_m couples, each one containing a Log-Likelihood Ratio, L_i, and its associated GF(q) symbol, a_i, i.e. $(L_i, a_i)_{i \in 1,\ldots,n_m}$ where $L_1 \leq L_2 \leq \ldots \leq L_{n_m}$. In the following, index k is omitted ($y = y_k$). Each L_i is defined as:

$$L_i = \ln\left(\frac{P(y|\tilde{a})}{P(y|a_i)}\right) = \frac{d^2(C(a_i), y_k) - d^2(C(\tilde{a}), y_k)}{2\sigma^2} \qquad (1)$$

where $\tilde{a}$ is the GF(q) symbol associated to the nearest point to y in the QAM constellation, i.e. the one that maximizes $P(y|a_i)$ for $i \in (1, \ldots, n_m)$. The Euclidean distance between two points in the signal space is represented by $d()$.

Demapper: The function of the demapper is to generate the intrinsic message for each received symbol y_k. For the sake of simplicity, let $d_i^2 = d^2(a_i, y)$ and $\delta^2 = d^2(\tilde{a}, y)$. Moreover, $d_i^2 = d_{iI}^2 + d_{iQ}^2$ and $\delta^2 = \delta_I^2 + \delta_Q^2$ as we decompose the M-QAM into two $2^{m/2}$-ary PAMs for distance calculation. Then, we can write:

$$\underbrace{d_i^2 - \delta_i^2}_{2\sigma^2 L_i} = \underbrace{(d_{iI}^2 - \delta_I^2)}_{\mathbf{U(i)}} + \underbrace{(d_{iQ}^2 - \delta_Q^2)}_{\mathbf{V(i)}} \qquad (2)$$

Finally, the objectif is to select the n_m smallest distances (sorted in increasing order) and their associated symbols a_i, in order to generate the intrinsic EMS message for y.

Finding the minimum distances with the Bubble-Check algorithm: In [10], the authors presented a low-complexity algorithm for extracting the n_m minimum values in the set defined as $U(i) + V(j)$, $(i, j) \in [1, n_m]^2$. This set is represented as a matrix T_Σ, where $T_\Sigma = U(i) + V(j)$. The elements in $\mathbf{U} = [U(1), U(2), \ldots, U(n_m)]$ and $\mathbf{V} = [V(1), V(2), \ldots, V(n_m)]$ are sorted in increasing order. Then, we can directly apply the Bubble-Check algorithm to generate the intrinsic message, as illustrated in Figure 2.

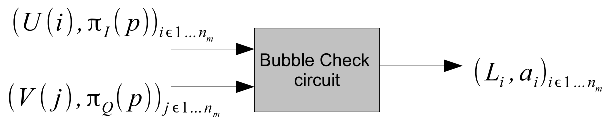

Fig. 2 *Application of the Bubble-Check circuit to generate the LLR intrinsic message*

Example for $M = q = 64$ and $n_m = 8$: Let us consider the case of a 64-QAM associated to a GF(64)-LDPC code with a Gray mapping as in the IEEE.802.11 standard (Fig. 4). Let $\mathbf{G}$ be $[-7, -5, -1, -3, 7, 5, 1, 3]$, then $\pi_I(p) = \mathbf{G}[\lfloor p/8 \rfloor]$ and $\pi_Q(p) = \mathbf{G}[p \mod 8]$. This way $C(a_{52}) = (\mathbf{G}(6), \mathbf{G}(4)) = (+1, +7)$ or $C(a_{32}) = (\mathbf{G}(4), \mathbf{G}(0)) = (+7, -7)$.

Let us now illustrate the demapping with the example that the received noisy signal is $y = (5.3, -3.2)$. Then, $C(\tilde{a}) = (+5, -3)$ and $\delta^2 = 0.3^2 + 0.2^2$. Let us focus first on the I axis. The calculation of the sorted values in $U(i)$ can be performed with a state machine (Figure 3 and Table 1) to obtain:

$$(\mathbf{U}(i), \pi_I(p)) = \{(0, +5), (2.8, +7), (5.2, +3), (18.4, +1), \ldots\} \qquad (3)$$

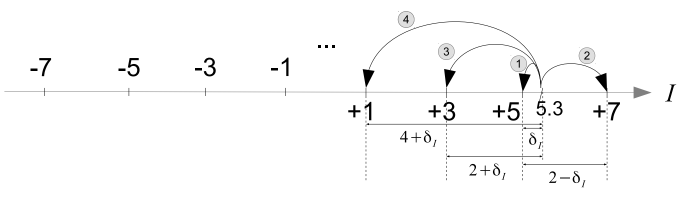

Fig. 3. Sequential distance computation on the I axis

The same procedure for the Q axis generates:

$$(\mathbf{V}(j), \pi_Q(p)) = \{(0, -3), (3.2, -5), (4.8, -1), (14.4, -7), \ldots\} \qquad (4)$$

Finally, Table 2 illustrates the generation of the EMS intrisic message $(L_i, a_i)_{i \in (1,\ldots,n_m)}$ through the Bubble-Check circuit (Fig. 2). For $n_m = 8$ the intrinsic message is $(0, a_{43}), (2.8, a_{35}), (3.2, a_{41}), (4.8, a_{42})$, $(5.2, a_{59}), (6, a_{33}), (7.6, a_{34}), (8.4, a_{57})$ which corresponds to the n_m closest signals to y.

Table 1: Distance computation on the I axis

i	$\pi_I(p)$	$\lfloor p/8 \rfloor$	d_{iI}^2	$\mathbf{U}(i)$
1	+5	5	δ_I^2	0.0
2	+7	4	$4+\delta_I^2-4\delta_I$	2.8
3	+3	7	$4+\delta_I^2+4\delta_I$	5.2
4	+1	6	$16+\delta_I^2+8\delta_I$	18.4
...	...	...	...	...

Table 2: Application of the Bubble-Check algorithm

	$(\mathbf{U}(i),\,\pi_I(p))$			
$(\mathbf{V}(j),\,\pi_Q(p))$	(0, +5)	(2.8, +7)	(5.2, +3)	(18.4, +1)
(0, -3)	$(L_1=0, a_{43})$	$(L_2=2.8, a_{35})$	$(L_3=5.2, a_{59})$	18.4
(3.2, -5)	$(L_3=3.2, a_{41})$	$(L_6=6.0, a_{33})$	$(L_8=8.4, a_{57})$	21.6
(4.8, -1)	$(L_4=4.8, a_{42})$	$(L_7=7.6, a_{34})$	10.0	23.2
(14.4, -7)	14.4	17.2	19.6	32.8
...	...	...	...	...

Results: Fig. 5 presents the simulation results obtained for ultra-sparse protograph-based NB-LDPC on GF(64) associated to a 64-QAM as in Fig. 4 for frame sizes of $N=192$ symbols (1152 bits) and $N=384$ (2304 bits) with a code rate of 1/2 over the AWGN channel. The BP curves correspond to the Belief Propagation decoding, simulated on floating point with 100 decoding iterations (see [11]). The EMS curves consider the EMS NB-LDPC decoder described in [12] with $n_m=12$, 20 decoding iterations, 6-bit quantization [2] and the intrinsic LLR generation presented in this Letter. A performance gap of about 0.4 dB is observed between the BP and the EMS curves, which confirms the interest of our approach in both performance and low-complexity implementation aspects.

Conclusion: This Letter focusses on low-complexity intrinsic LLR generation for high-order NB coded QAM designs. The originality remains in the use of the Bubble-Check algorithm for the computation of the intrinsic message. The simulation results show the interest of this work in terms of perfomance. The FPGA implementation of the NB-LDPC EMS decoder and the Bubble-Check architecture design considered in [12] are the proof of the implementation feasibility of both the QAM demodulator and decoder (Fig. 1).

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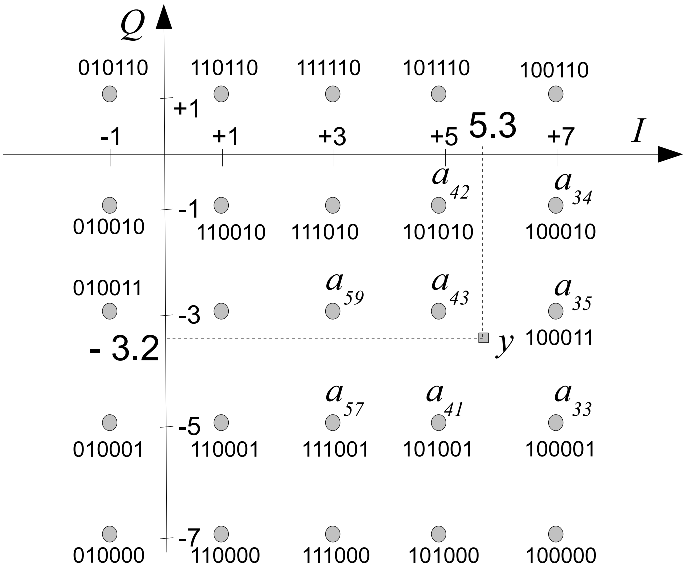

Fig. 4 *Zoom on the 64-QAM constellation and the n_m closest points to y for the EMS intrinsic message generation*

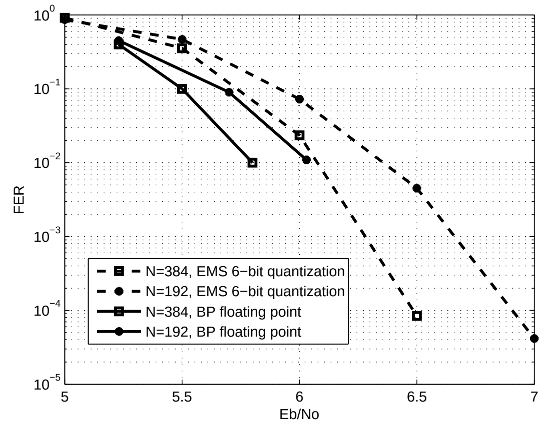

Fig. 5 *Performance comparison of the BP and EMS for a GF(64)-LDPC associated to a 64-QAM*

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[2] Note that this decoder design was implemented on FPGA [12]